Performance Task: Working Two Jobs

Tom is currently working two jobs. He is limited in how many hours in a given week he can work both jobs. The shading in the graph below is known as the “feasible region”, in that it represents all of the possible combinations of hours Tom could work in each job (job A represented along the *x*-axis and job B represented along the *y*-axis).

**Part A**

Each line that is graphed represents a constraint. For each equation, describe in the context of the problem what the constraints might represent.

 *B*

 🡨 $A+B\leq 50$

 🡨 $B\geq A$

 🡨 $B\leq 30$

 🡨 Feasible 🡨Feasible Region

 *A*

$A\geq 0$:

$B\geq 0$:

$B\leq 30$:

$B\geq A$:

$A+B\leq 50$:

**Part B**

Tom’s two jobs each pay a different rate per hour. Job A pays $15 an hour and Job B pays $20 an hour. Staying within the feasible region, what combination of hours will provide Tom with the most money? This question can be answered using a field of study in mathematics called “linear programming”. The theory is that the profit can be maximized or minimized only at one of the points at a corner of the region.

1. Using the graph and equations as a reference, determine the coordinates of the points at the corners of the feasible region.
2. For each of these points you came up with, determine which combination will maximize the profit for Tom. How many hours should he work at each job? What will his weekly income be?

Performance Task Overview: Working Two Jobs

TASK PERFORMANCE LEVELS:

Level 1: Demonstrates Minimal Success (0-7 points)

The student’s response shows few of the elements of performance that the tasks demand. The

work shows a minimal attempt on the problem and struggles to make a coherent attack on the

problem. Communication is limited and shows minimal reasoning. The student’s response

rarely uses definitions in their explanations. The student struggles to recognize patterns or the

structure of the problem situation.

Level 2: Performance below Standard (8-15 points)

The student’s response shows some of the elements of performance that the tasks demand and

some signs of a coherent attack on the core of some of the problems. However, the

shortcomings are substantial and the evidence suggests that the student would not be able to

produce high-quality solutions without significant further instruction. The student might ignore

or fail to address some of the constraints. The student may occasionally make sense of

quantities in relationships in the problem, but their use of quantity is limited or not fully

developed. The student response may not state assumptions, definitions, and previously

established results. While the student makes an attack on the problem, it is incomplete. The

student may recognize some patterns or structures, but has trouble generalizing or using them

to solve the problem.

Level 3: Performance at Standard (16-22 points)

For most of the task, the student’s response shows the main elements of performance that the

tasks demand and is organized as a coherent attack on the core of the problem. There are

errors or omissions, some of which may be important, but of a kind that the student could well

fix, with more time for checking and revision and some limited help. The student explains the

problem and identifies constraints. The student makes sense of quantities and their

relationships in the problem situations. The student often uses abstractions to represent a

problem symbolically or with other mathematical representations. The student response may

use assumptions, definitions, and previously established results in constructing arguments. They

may make conjectures and build a logical progression of statements to explore the truth of their

conjectures. The student might discern patterns or structures and make connections between

representations.

Level 4: Achieves Standards at a High Level (23-30 points)

The student’s response meets the demands of nearly the entire task, with few errors. With

some more time for checking and revision, excellent solutions would seem likely. The student

response shows understanding and use of stated assumptions, definitions and previously

established results in construction arguments. The student is able to make conjectures and build

a logical progression of statements to explore the truth of their conjecture. The student

response routinely interprets their mathematical results in the context of the situation and

reflects on whether the results make sense. The communication is precise, using definitions

clearly. Students look closely to discern a pattern or structure. The body of work looks at the

overall situation of the problem and process, while attending to the details.

COMMON CORE STANDARDS ASSESSED:

A-CED.1, A-CED.3, A-REI.3, A-REI.6, A-REI.12, F-BF.1,

ANSWER KEY:

 **Part A (10 points)**

 $A\geq 0$: Tom can’t work negative hours at Job A.

 $B\geq 0$: Tom can’t work negative hours at Job B.

 $B\leq 30$: Tom can’t work more than 30 hours at Job B.

 $A+B\leq 50$: Tom can’t work more than a total of 50 hours at both jobs.

 $B\geq A$: Tom can’t work more hours at Job A than Job B.

**Part B (20 points)**

1. The corners of the feasible region are located at (0,0), (0, 30), (25, 25) and (20, 30).
2. $P=15A+20B$

(0, 0): $P=\$0$

(0, 30): $P=\$600$

(25, 25): $P=\$875$

(20, 30): $P=\$900$

Tom should work 20 hours at Job A and 30 hours at Job B. His weekly income will be $900.

RUBRIC:

|  |  |
| --- | --- |
| Elements of the Task Include:* Connecting Equations and Graphs of Inequalities in Two Variables to a Context
* Solve Systems of Equations in Two Variables
* Develop an Equation from a Context
* Evaluate an Equation and Interpret the Results
 | Points Earned |
| **Part A (10 points)****5 points:** Describe a reasonable connection to the hours worked at each job to each inequality. The terms “at least” and/or “at most” should be used.**2 points:** Show evidence that the student understands the domain of the context cannot include negative values for either A or B.**3 points:** Describe the inequality that includes both A and B as a situation in which the amount of hours worked at each job effect one another. |  |
| **Part B (20 points)****6 points:** Identify the intersections of each of the linear equations by using either a graphical or algebraic approach.**10 points:** Develop a model for the total weekly income or understand the appropriate algorithm for its calculation. Apply the formula/algorithm to determine the total weekly income.**4 points:** State answer in relationship to the context of the problem, and label with appropriate units.  |  |